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Article

Multi-stage Roundabout Production

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Abstract

This paper presents a model of multi-stage roundabout production with tiered structures. The model assumes one original factor, called labor, allocated to multi stages/sectors of production. They are vertically and sequentially linked such that the first sector produces a primary intermediate good, the second sector a secondary intermediate good, the third and so on sectors produce likewise intermediate goods, towards the final sector that produces the final good for consumption. We address the problem of how many stages/sectors of roundabout production would be optimal for maximizing output and derive related conditions under which roundabout production continues or discontinues.

Keywords: Multi-stage production, roundabout production, intermediate goods, vertically related sectors, sectoral factor intensities

1. Introduction

Methods of production are roundabout if they use the output of primitive stages (or low-tier sectors) as inputs for later stages (or high-tier sectors). When stages of production are linked sequentially, the process is that of multi-stage roundabout production (or more commonly called multi-stage production). The notion of roundaboutness in production is

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not new. Dating back to the eighteenth century, in British classical economics (e.g., Adam Smith) the degree of roundaboutness in production was linked to the division of labor: Implied in higher degree of roundaboutness were an increase in the division of labor and a rise in productivity. A century later, Böhm-Bawerk (1884), enriched the tradition by bringing in time-consuming nature of roundabout production, as influenced by Austrian capital theory. (See Hennings (1987) for more detail.) Time passed and since around the turn of the last century, the idea of the division of labor and the meaning of varieties of intermediate inputs associated with it have been attracted an academic attention, both in the trade and growth literature.[1)] In retrospect, the focus of research has shifted from studying the functioning of roundabout production itself to looking at intermediate goods produced as a source for sustained growth and gains from trade. Along the line, particular settings have been introduced: monopolistic competition and increasing returns associated with increasing number of intermediate goods.

Departing from the above literature, the present study focuses on the functioning of roundabout production itself under normal, standard assumptions. As usual, constant returns to scale and diminishing returns to factor are assumed because our goal is not to explain sustained growth observable at the aggregated or country level, but rather to understand roundaboutness in production conceivable at the industry or firm level, where the standard assumptions are more appropriate. Furthermore and importantly, we consider multi-stage or n-stage production that has not been considered in the trade/growth literature (where only one stage of production using intermediate goods is assumed).[2)] The n-stage case, as will be shown, is not an obvious extension of 2-stage case, particularly when factor intensities (i.e., output elasticities to labor and to inter-

1) An early study that associated increasing returns with the increased specialization or division of labor can be traced to Allyn Young (1928). The idea of the specialization has been incorporated both into international trade theories (Ethier, 1982) and endogenous growth theories (Romer, 1990).

2) More recently two-stage production (i.e., the second stage of production uses the output of the first stage) has been considered in the trade literature (e.g., Yi, 2010).

mediate input) differ across stages/sectors. Multi-stage production also has much real-world relevance as seen in many industries including agriculture, manufactures, services, and so forth.[3] However, the theoretical investigation has not been sufficiently carried out. We fill in the gap.

Our model, as motivated by Böhm-Bawerk, features multiple tiered-production structure where stages/sectors of production are sequentially linked.[4] With such structure of production, roundaboutness (or the number of stages) of production is endogenous; that is, the optimal resource allocation to different stages/sectors of production is determined endogenously.[5] We ask under what conditions roundabout production would be feasible, and find out the optimal number of stages/sectors of roundabout production and the conditions associated with it.

In what follows Section 2 presents a simple model of roundabout production *a la* Böhm-Bawerk, or 2-stage roundabout production with a given endowment, notably labor, as the sole original factor of production. The simple model *a la* Böhm-Bawerk can be featured with multi-stages/sectors (or n-stages/sectors) of production. Section 3 deals with a higher degree of roundaboutness in production and find out under what condition roundabout production (designated as RAP) continues or discontinues. The last section concludes.

2. Roundabout production *a la* Böhm-Bawerk: 2-stage RAP model

Consider first a simple 2-stage (or sector) model of Böhm-Bawerkian roundabout production. Here the initial-stage (or primitive sector) of production requires labor as a sole original factor of production. The output from the primitive sector is assumed to be used in either one of

3) As far as the 2-stage case is concerned, empirical studies have been conducted; e.g., Pollak and Wales (1995) work on empirical production analysis with a 2-stage setup.

4) Greenhut and Ohta (1976, 1979) address to seemingly similar models with vertically related intermediate sectors, similar in the present model, but *non-sequitur*. They in fact are diametrically different from ours in scope and conclusions.

5) This vertically related process of production contrasts sharply with a horizontal (or flat) production process where varieties of intermediate goods enter the production function of final goods in a symmetric way (as seen in the trade/growth literature), implying identical intermediate goods being produced in the same amount.

the following ways: a) consumed entirely as a final consumption good or b) used entirely as an intermediate good (to be combined with labor) in the next-stage production of a final consumption good. In this structure labor is allocated over the two stages/sectors; workers are assumed to be employed in either one sector only, but not in both sectors simultaneously; labor is hence to be allocated in a sequential manner.[6] These in a nutshell are the crucial aspect of the roundabout methods of production *a la* Böhm-Bawerk, Eugen v. (1884).[7]

The structure of roundabout production described above may be formally presented as follows. (For expositional purposes, labor endowment and technology parameters are all normalized to one.)

$$\text{2–1)} \quad Y = X^{\alpha} L_Y^{1-\alpha} = [\delta(1-L_Y)]^{\alpha} L_Y^{1-\alpha}$$

$$\text{2–2)} \quad X = \delta(1-L_Y)$$

where 2–2) defines the first stage/sector of production where *primary* intermediate goods X is produced for input in the second stage/sector to produce final goods Y for consumption. This final stage production may be given, for simplicity, by the specific Cobb-Douglas form of production function 2–1) above. The underlying assumptions here include a given endowment of labor L (=1) as *original* factor of production, part of which, $1-L_Y$, is used to produce the primary intermediate goods X with a given labor productivity of δ. This $X = \delta(1-L_Y)$, in turn, combined with the labor L_Y still available upon labor input L_X (=$1-L_Y$) to produce X, must yield the final consumption goods. Given the second line specification of 2–1), note that the final good Y depends on one single *endog-*

6) A multistage production process using labor and output from the precedent stage is ubiquitous: software development, craft-man's work, basic research (including a process we bitterly experience while writing/editing research papers ☺), and so forth. After all, our work is built on precedent work of our own and/or others.

7) Contributions of Böhm-Bawerk arose a renewed interest of later scholars — including Shibata (1935, 1959), whose contribution to theoretical economics has long been forlorn in the literature, despite a candid appraisal by Oscar Lange (1935). To quote from Lange (p. 189) "Shibata has performed an exceedingly fine piece of analysis for which any serious economist should be grateful."

enous variable L_Y other than two *exogenous* parameters, δ and α. Also note that both production functions are subject to constant returns to scale.[8)]

Consider now the first-order condition, FOC, to maximize Y with respect to L_Y:

$$\mathrm{MP}_{\mathrm{LC}} = 0$$
$$\therefore L_Y = 1-\alpha$$

Substituting this back in 2–2) above yields:

$$\text{2–3)} \quad Y^* = \delta^{\alpha}(1-L_Y)^{\alpha} L_Y^{(1-\alpha)} = \delta^{\alpha}\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}$$

This is the reduced form solution to maximum output of final goods in terms of δ and α. A couple of notes are warranted. First, when labor alone exists as the original factor of production, there exists a certain threshold level of labor productivity above which roundabout methods of production may be aborted to begin with. The final-stage output Y^* would be strictly greater than that obtained under direct production when $Y^* = \delta^{\alpha}\alpha^{\alpha}(1-\alpha)^{(1-\alpha)} > \delta$, that is:

$$\text{2–4)} \quad \delta < (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}$$

This condition implies that unless labor productivity δ is low enough to be lower than the right-hand-side of the inequality above, roundabout production becomes meaningless, as it yields an inadvertent outcome lesser than does direct, non-roundabout production. To get a feel for this condition (which also becomes important when we later consider RAP3 and beyond), see Figure 1 displaying how final output of Ys from roundabout production depends on α for alternative non-roundabout labor productivities of and confirm that each horizontal line segment corresponds to a threshold output level below which RAP is infeasible.

Second, also inferred from in Figure 1, the smaller the α (i.e., the

8) If instead equation 2–2) were specified as $X = (1-L_Y)^{1-\alpha}$, there would be no roundabout production to begin with.

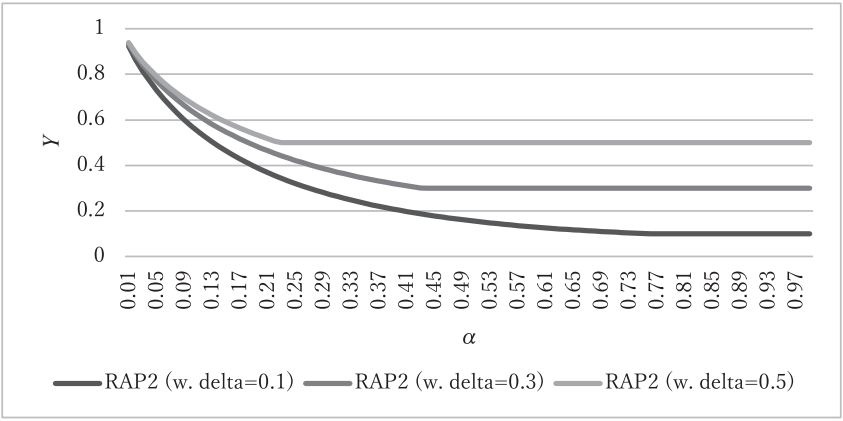

Figure 1. RAP2 final output Ys and α (for δ=0.1, 0.3, 0.5)

higher the output elasticity of labor input 1–α) in the final consumption good sector, the higher the optimal labor input therein, thereby implying the higher labor intensity and the lesser intermediate good or capital intensity of that sector. Thus, while the intermediate capital goods are indispensable for RAP, the outcome of production in the final consumption good sector may be better, the smaller (rather than larger) the ratio of intermediate goods input per labor therein provided that (final) output elasticity of labor is large enough; the larger, the better indeed.[9] (Akiba, 2005; Ohta, 2008.)

In light of the implications of the 2-stage RAP model above, we may next ask: if going roundabout increases final output, does going further around, i.e., roundabout, by increasing the number of intermediate sectors also increase productivity? If so, under what conditions? To answer

9) For a proof consider roundabout final output Y^* divided by direct output of δ. Let $y^*= Y^*/\delta$. Then

$$y^* = \delta^{\alpha-1}\alpha^{\alpha}(1-\alpha)^{(1-\alpha)} = \alpha^{\alpha}\left(\frac{1-\alpha}{\delta}\right)^{1-\alpha}.$$

$$\therefore(\delta y^*/\delta\alpha) = y^* \ln \delta\alpha/(1-\alpha)<0 \quad \forall\delta<(1/\alpha)-1.$$

Hence, for any given $\delta<(1/\alpha)-1$, it follows that the smaller the α is, the larger (not smaller), the y^* (or Y^*) increasingly higher than δ.

these questions, we now consider in the next section an alternative 3- as well as n-stage/sector model of roundabout production to be compared with the one with 2 stages/sectors considered above.

3. RAP model: 3-stage RAP and beyond

3. 1 Case 1: same α for all stages of RAP

Our three-stage/sector model of roundabout production, or RAP3, is given as follows.

3–1) $Y_3 = X_2^{\alpha} L_Y^{1-\alpha}$
3–2) $X_2 = X_1^{\alpha} L_{X2}^{1-\alpha}$
3–3) $X_1 = \delta L_{X1}$
3–4) $L_{X1} = (1-L_Y-L_{X2})$

where X_2 stands for *secondary* intermediate capital good (or second-stage output), X_1 for *primary* intermediate capital good (or first-stage output), and L_{X2} for labor input to the *secondary* intermediate capital good X_2 sector, respectively. Note that the *secondary* intermediate good as input is distinguished from the *primary* intermediate factor of production. But labor alone is the original factor of production which transforms itself, however, under the RAP scheme into the primary, secondary, tertiary, and so on intermediate factors of production.

By backward substitution to obtain Y as a function of L_Y and L_{X2}: $Y = Y(L_Y, L_{X2})$, the relevant FOC with respect to L_Y and L_{X2} yields:

3–5) $$\frac{\delta Y}{\delta L_Y} = 0: \frac{\alpha^2 L_Y}{1-L_Y-L_{X2}} = 1-\alpha$$

3–6) $$\frac{\delta Y}{\delta L_{X2}} = 0: \frac{\alpha L_{X2}}{1-L_Y-L_{X2}} = 1-\alpha$$

The above system of equations contains 6 unknown variables, Y, L_Y, L_{X2}, L_{X1}, X_2, and X_1. The solution is then expressed in terms of one single parameter α as:

$$L_Y^* = 1-\alpha$$
$$L_{X2}^* = \alpha(1-\alpha)$$

$L_{X1}^{*} = \alpha^2$ inasmuch as $L_{X1} = 1 - L_Y - L_{X2}$

3–7) $Y_3^{*} = Y^{*}(\text{RAP3}) = \delta^{\alpha^2}\alpha^{\alpha(1+\alpha)}(1-\alpha)^{(1-\alpha)(1+\alpha)}$

Now RAP3 can be compared with RAP2 outcome obtained in the last section. Of our particular interest is whether the final output of RAP3 exceeds that of RAP2 (that is, whether the second round of roundabout production is feasible, as implied by $Y_2^{*} < Y_3^{*}$). It is revealed that there will be the second round (or RAP3) provided that there was the first round of roundabout production (or RAP2). This claim is confirmed by the inequality:

$$\frac{Y_3^{*}}{Y_2^{*}} = \left[\delta^{\alpha-1}\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\right]^{\alpha} > 1 \text{ provided that } \delta < \delta^{\alpha}\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}.$$

The proviso above is equivalent to condition (2–4) — the condition required for RAP2 to be feasible.

Therefore, the result here is as follows:

> If the first round of roundabout production is feasible, so is the next round of roundabout production. More generally, for further roundabout production such as RAP4, RAP5, and so forth, it can be seen that the process of roundabout production continues; the output level increases monotonically and converges to the level corresponding to RAP*n* with n→∞. Naturally, a flip side of this is that if the first round of roundabout production is infeasible, the direct production is all there is; no further roundabout productions occur.

3. 2 Case 2: different *α's* for each stage of RAP

Here we extend the above analysis to a more general case: factor intensities of each stage/sector are allowed to differ. Specifically, replacing equations 3–1) and 3–2) earlier, we set the production functions for the final good and intermediate good sectors:

3–1)′ $Y_3 = X_2^{\alpha_2} L_Y^{1-\alpha_2}$

3–2)′ $X_2 = X_1^{\alpha_1} L_{X2}^{1-\alpha_1}$,

where $\alpha_1 \neq \alpha_2$. The rest of the setup is the same as before (as seen in

equations 3–3) and 3–4)). The modified specification yields the maximized level of final output:

3–7)′ $Y_3^*(\text{RAP3}) = \left[\delta^{\alpha_1}\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\right]^{\alpha_2}\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}$.

We now obtain the following result for RAP3 (provided condition (2–4)).[10)]

In the case of $\alpha_1>\alpha_2$, RAP3 is feasible. In fact, $\alpha_1>\alpha_2$ is a sufficient condition for RAP to be feasible.[11)]

In the case of $\alpha_1<\alpha_2$, there exist a large enough α_2 such that RAP3 is infeasible; production beyond RAP2 is infeasible.

The result for RAPn is obtained analogously.[12)]

In the case of $\alpha_1>\alpha_2>\alpha_3>\ldots>\alpha_{n-1}$, RAP$n$ is feasible.

For other cases, even if the feasibility of preceding stages of production were met, for a given large enough α_{i-1}, RAP of i-th stage and beyond is infeasible.

In considering above, the economic intuition can be understood from the insight drawn from RAP2 in Section 2: the larger the output elasticity of labor $1-\alpha$, the better indeed for the final output. What may be counter-intuitive is that while intermediate capital good is needed for roundabout production, labor-intensive production is in fact blessing for roundabout production; as more sectors and stages of production arise, labor-deepening progresses — quite a contrast to a view that they arise con-

10) The result here is for RAP3 with $\alpha_1\neq\alpha_2$. For $\alpha_1=\alpha_2$, the result is contained in Section3.1.

11) To be more exact, the necessary and sufficient condition for RAP3 to be employed is:

$\delta^{\alpha_1}\alpha_1^{\alpha_1}(1-\alpha_1)^{(1-\alpha_1)}<\alpha_2^{\frac{\alpha_2}{1-\alpha_2}}(1-\alpha_2)$. This is obtained from $\dfrac{Y_3^*{}_{\alpha_1,\alpha_2}(\text{RAP3})}{Y_2^*{}_{\alpha_1}(\text{RAP2})}>1$, where $Y_2^*{}_{\alpha_1}(\text{RAP2}) = \delta^{\alpha_1}\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}$.

12) The n-stage setup can be expressed more generally and succinctly as:

$Y_n = X_{n-1}^{\alpha_{n-1}}L_Y^{1-\alpha_{n-1}}$.
$X_i = X_{i-1}^{\alpha_{i-1}}L_{Xi}^{1-\alpha_{i-1}}$ for $i = 2,3..\ ..n-1$
$X_1 = \delta L_{X1}$
$L_{X1} = (1-L_Y-\sum_{i=2}^{n-1}L_{Xi})$

currently with capital-deepening.

4. Concluding Remarks

We have proposed a model describing multi-stage/sector roundabout production (RAP) and presented how the degree of roundaboutness is determined by the factor intensity of each stage/sector involved in the RAP process. Interestingly, it is shown that, while intermediate capital good is a requisite input for roundabout production, more labor-intensive stages/sectors of production (rather than more capital-intensive ones) facilitate RAP to raise final output level. The intuition is that the curse of diminishing returns to labor is magnified each time the stage-specific intermediate capital goods is used for RAP as an input.

For future inquiries, more explicit dynamics can be incorporated into the model. At present, the time element is recognized only to the extent that stages/sectors are sequentially linked. To be more satisfactory, one may introduce intertemporal choice of consumption/investment: intermediate capital goods are either consumed or invested as input for the next stage RAP. With the intertemporal feature, our understanding of roundabout production along with other dynamic models can be enhanced.

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